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Mathematical Analysis of Problems in Turbulence and Turbulent Diffusion with Many Statistical Scales

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13. ABSTRACT *(Maximum 200 words)*

The research program being funded here emphasizes problems in turbulence and turbulent diffusion which are inherently statistical and involve many spatio-temporal scales. One goal of the research is to achieve a better theoretical understanding of turbulent (reaction) diffusion which is crucial for many applications in environmental science and engineering such as the tracking of pollutants in the atmosphere, the behavior of chemical tracers in the ocean and porous media, and turbulent combustion. Other parts of the research emphasize the interaction and generation of both small scales and large scale coherent structure in various anisotropic turbulent flows from both a statistical and deterministic point of view. The approach to all of these issues involves a combination of asymptotic analysis, numerical computation, and theoretical analysis to gain insight into these complex and important phenomena.

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1. List of Manuscripts

Papers Published

1. "Simplified Asymptotic Equations for Slender Vortex Filaments", *Proc. of Symposia in Applied Math*, 54, pp. 237-280, 1998

2. (with M. Holen), "Dissipation, topography, and statistical theories for large scale coherent structure", *C.P.A.M.*, Volume L (12), pp. 1183-1234, 1997.

3. (with P. Constantin, C. Foias, and I. Kukavica), "Dirichlet quotients and 2-D periodic Navier-Stokes equations", *Math. Pures and Appl.*, 76, pp. 125-153, 1997

4. (with F. W. Elliott, Jr. and D.J. Horntrop), "A Fourier-Wavelet Monte Carlo Method for fractal random fields", *J. Comp. Physics,* 132, pp. 384-408, 1997

5. (with D. McLaughlin and E. Tabak), "A one-dimensional model for dispersive wave turbulence", *J. Nonlinear Sci* 6, pp. 9-44, 1997

6. (with F. W. Elliott, Jr. and D.J. Horntrop), "Monte Carlo Methods for turbulent tracers with long range and fractal random velocity field", *Chaos* 7 (1), pp. 39-48, 1997

7. (with M. Grote), "Model Dynamics and Vertical Collapse in Decaying Strongly Stratified Flows", *Phys. Fluids* 9 (10) pp. 2932-2940, 1997

8. (with M. Grote), "Crude closure dynamics through large scale statistical theories", *Phys. Fluids* 9 (11), pp. 3431-3442, 1997

9. (with R. McLaughlin), "An explicit example with non-Gaussian probability distribution for nontrivial scalar mean and fluctuation", *Phys. Fluids*, 8 (2), pp. 536-547, 1996

10. (with F. Elliott), "Pair dispersion over an inertial range spanning many decades", *Phys. Fluids*, 8 (4), pp. 1052-1060, 1996

11. (with P. Embid), "Averaging over fast gravity waves for geophysical flows with arbitrary potential vorticity", *Comm. P.D.E.*, 21 (3&4), pp. 619-658, 1996

12. (with P. Constantin and C. Fefferman), "Geometric constraints on potentially singular solutions for the 3-D Euler equations", *Comm. P.D.E.*, 21 (3&4), pp. 559-571, 1996

13. (with T. Souganidis), "Bounds on enhanced turbulent flame speeds for combustion with fractal velocity fields", *J. Stat. Phys.*, 83 (5&6), pp. 933-954, 1996

14. (with E. Tabak), "A Two-dimensional model for quasigeostrophic flow. Comparison with two-dimensional Euler", *Physica D*, 1413, pp. 1-8, 1996

15. (with P. Embid), "Examples and counterexamples for Huygens Principle in premixed combustion", *Comb. Sci and Tech.*, 120 (1-6), pp. 273, 1996

Papers Submitted and/or Accepted

1. (with P. Souganidis), "Flame fronts in a turbulent combustion model with fractal velocity fields", to appear in C.P.A.M. (Fritz John Volume).

2. (with P. Embid), "Low Froude Number limiting dynamics for stably stratified flow with small or fixed Rossby Numbers", to appear in *G.A.F.D.*

3. (with P. Embid) "Averaging over fast gravity waves for geophysical flows with unbalanced initial data", to appear in *Theor. Comp. Fluid Dyn.*

4. (with D. Horntrop) "An overview of Monte Carlo simulation techniques for the generation of random field", to appear in *Proceedings of the 9th Aha Huliko Hawaiian Winter Workshop*

5. (with M.J. Grote and C.G. Ragazzo) "Dynamic mean flow and small scale interaction through topographic stress", to appear in *J. Nonlinear Sci.*

6. (with E. Tabak, R. Rosales, and C. Turner), "Interaction of large scale equatorial waves and dispersion of kelvin waves through topographic resonances", submitted to *J. Atmos. Sci.*

7. (with M. DiBattista and B. Turkington), "Prototype geophysical vortex structures via large-scale statistical theory", submitted to *G.A.F.D.*

2. Scientific Personnel

David Horntrop, postdoctoral fellow, was supported during this reporting period.
Marcus Grote, postdoctoral fellow, was supported during this reporting period.

3. Report of Inventions

none

4. Scientific Progress and Accomplishments

During the period of the grant, one large effort by Majda involved novel wavelet methods for Monte Carlo simulation of turbulent diffusion with random velocity fields having many spatial scales. Majda and Elliott [(10)] have applied these methods to verify Richardson's t^3 law for pair dispersion over *eight decades of separation* with an accurate preconstant over these same eight decades with the velocity field defined through Taylor's hypothesis; these are the first simulations checking Richardson's law for more than one decade of separation. Majda has worked with his post doc, Horntrop, on Fourier-based wavelet Monte Carlo methods for turbulent transport which are more flexible and suitable for random fields generated by waves. Horntrop and Majda have also used exactly solvable models to demonstrate subtle numerical issues with conventional Monte Carlo algorithms ([4], [6]; to appear [4]). During this granting period, Majda and his research group have also established, both rigorously and practically, that the "successive random addition algorithms" for generating fractal random fields, which are very popular in the physics community, cannot be consistent with a stationary Gaussian fractal field.

Another major effort by Majda and his collaborators has been to provide the first unambiguous check of the G-closure procedure for turbulent combustion as compared with the rigorous renormalization theory developed recently by Majda and Souganidis. For small scale turbulence defined by shear flows, there are both regimes of success for the G-closure method and also regimes of strong underprediction of the flame speed through G-closure. Majda and Souganidis ([13]) have developed the first upper bounds for turbulent combustion with fractal velocity fields. Also, Majda, Embid and Souganidis ([15]) have shown through systematic unambiguous models that the Huygen's principle for large scale flame propagation can be violated strongly even for F-K-Z chemistry at high activation energies; the true large scale flame front dynamics are governed instead by a variational inequality, as developed earlier by Majda and Souganidis for K-P-P chemistry.

As regards fluid dynamics, Majda has continued his work on singular solutions for the 3-D Euler equations as well as related models ([12], [14]). With his post doc, Marcus Grote, he has also developed new model dynamics which display vertical collapse in strongly stratified flows ([7], [8]). Majda has also developed the first models with D. McLaughlin and E. Tabak where dispersive wave turbulence can be checked in an unambiguous fashion.

CURRICULUM VITAE
Andrew J. Majda

EDUCATION:

B.S.	Purdue University, 1970
M.S.	Stanford University, 1971
Ph.D.	Stanford University, 1973

EMPLOYMENT:

1973-76	New York University, Courant Institute of Mathematical Sciences, Instructor
1976-77	University of California, Los Angeles, Assistant Professor
1977-78	University of California, Los Angeles, Associate Professor
1978, Fall	University of California, Los Angeles, Professor
1979, Jan. – Sept. 1984	University of California, Berkeley, Professor
1984 to Sept. 1995	Princeton University, Professor
1994, Sept.	New York University, Samuel Morse Professor of Arts & Sciences at Courant Institute of Mathematical Sciences

HONORS RECEIVED:

1977 - 79	Alfred P. Sloan Foundation Fellow
1982, November	Medal of the College de France
1990, July	John von Neumann Award of SIAM and delivered John von Neumann Lecture at SIAM National Meeting, Chicago
1992, April	National Academy of Sciences Prize in Applied Mathematics and Numerical Analysis
1994, April	Elected to the National Academy of Sciences
1995, January	Gibbs Lecturer of the American Mathematical Society

MAJOR INVITED ADDRESSES

1983, August	International Congress of Mathematics, Warsaw
1984, January	National American Mathematical Society Meeting, Louisville
1984, July	S.I.A.M. National Meeting, Palo Alto
1987, June	Plenary Address at First International Congress of Industrial and Applied Mathematics, Paris
1988, August	Centennial Celebration of American Mathematical Society, Providence, RI
1990, August	Plenary One Hour Address at International Congress of Mathematicians, Kyoto, Japan
1995, January	Gibbs Lecturer of the American Mathematical Society

PROFESSIONAL MEMBERSHIPS

National Academy of Sciences
American Mathematical Society
American Physical Society
Society for Industrial and Applied Mathematics

Recent Published Papers

1. (with F. Elliott), "A new algorithm with plane waves and wavelets for random velocity fields with many spatial scales", *J. Comp. Physics,* 117, pp. 146-162, 1995

2. (with A. Bourlioux), "Theoretical and numerical structure of unstable detonations", *Philos. Trans. Roy. Soc. Ser. A*, 350, pp. 29-68, 1995

3. (with R. Klein and K. Damodaran), "Simplified equations for the interaction of nearly parallel vortex filaments", *Journal of Fluid Mechanics*, 228, pp. 201-248, 1995

4. (with P. Embid and T. Souganidis), "Effective geometric front dynamics for premixed turbulent combustion with separated velocity scales", *Combustion Science and Technology*, 103, pp. 85-115, 1994

5. (with F. Elliott, Jr., D. J. Horntrop, and R. McLaughlin), "Hierarchical Monte Carlo methods for fractal random fields", *J. Statistical Physics*, 81, pp. 717-736, 1995

6. (with P. Embid and P. Souganidis), "Comparison of turbulent flame speeds from complete averaging and the G-equation", *Phys. Fluids A*, 7, pp. 2050-2060, 1995

7. (with R. McLaughlin), "An explicit example with non-Gaussian probability distribution for nontrivial scalar mean and fluctuation", *Phys. Fluids*, 8 (2), pp. 536-547, 1996

8. (with F. Elliott), "Pair dispersion over an inertial range spanning many decades", *Phys. Fluids*, 8 (4), pp. 1052-1060, 1996

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19. (with M. Grote), "Crude Closure Dynamics Through Large Scale Statistical Theories", *Phys. Fluids*, 9 (11), 3431-3442, 1997

20. (with M. Holen), "Dissipation, topography, and statistical theories for large scale coherent structure", *Commun. Pure App. Math.*, Volume L (12), 1183-1234, 1997

21. "Simplified Asymptotic Equations for Slender Vortex Filaments", *Proc. of Symposia in Applied Math*, 54, 237-280, 1998

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6. (with M. DiBattista and B. Turkington), "Prototype geophysical vortex structures via large-scale statistical theory", submitted to *G.A.F.D.*

Collaborators Since 1990:

R. Almgren, R. Rosales, V. Roytburd, M. Avellaneda, A. Bourlioux, R. Klein, K. Lamb, P. Embid, R. McLaughlin, J. Hunter, P. Constantin, E. Tabak, P. Souganidis, G. Majda, Y. Zheng, F. Elliott, D. Horntrop, K. Damodaran, C. Foias, I. Kukavica, C. Fefferman, D. McLaughlin

Ph.D. Students Since 1990:

D. Stuart, A. Bertozzi, A. Bourlioux, R. McLaughlin, M. Grote, D. Horntrop, M. Holen